Louisiana Highway Research

QUALITY CONTROL ANALYSIS

PART III

CONCRETE & CONCRETE AGGREGATES

Louisiana Department of Highways – Research & Development Section

QUALITY CONTROL ANALYSIS
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CONCRETE AND CONCRETE AGGREGATES

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SYNOPSIS

This is the third and last report on the Quality Control Analysis of highway construction materials.

It deals with the statistical evaluation of data from several construction projects to determine the basic pattern of variability with respect to slump of concrete and concrete aggregates.

The analysis indicated 1) that the frequency distribution of most of the data tend to follow normal distribution; 2) that, in general there is considerable variation in concrete production from batch to batch; 3) that for fine aggregate, the stockpile component of variance (as determined by ANOVA) contributes more to the overall variance than samples within stockpile component; 4) that in the case of coarse aggregates, the samples within stockpile component show larger variance than between stockpile component; 5) furthermore, that there is pronounced variation in pavement thickness from contractor to contractor.

The analysis of data on asphaltic concrete characteristics indicated 1) that there is considerable variation in pavement density from contractor to contractor; 2) that by far the largest source of variation in discharge temperature is indicated by process component.

The study has also revealed how control charts can be used for control and acceptance of Portland cement concrete production.

INTRODUCTION

For specifications to be realistic and of any value, estimation of the variability concerning the measured characteristic is of prime importance. Most of our present day specifications have evolved through trial and error approach without any specific reference to the variability of the production process. It is the purpose of this investigation to obtain estimates of variability associated with the production of portland cement concrete.

Application of statistical quality control techniques to highway construction materials has gathered considerable momentum over the past few years. The method has many important applications including writing of specifications realistically and providing for sounder relation between engineering and production and between producer and consumer.

The construction of highway is comparable to any manufacturing industry where we have a manufactured product, the highway and a need for the control of its quality which is desired by both the manufacturer or the contractor and the purchaser or the State Highway Department.

Variability is inherent in all manufactured products, be it nuts, bolts, glass tubes, or fresh portland cement concrete. In the case of the latter, any or all of the following sources can produce variations in the final product:

1. Type of aggregate
2. Proportion of aggregate
3. Water-cement ratio
4. Proportion of air entraining agent
5. Other intangible sources

Variations are further introduced when the concrete is sampled and tested for a particular characteristic. Using statistical quality control procedures, a manufacturing process can be investigated for the range of values one can expect under existing conditions of men, machinery, and materials.

SCOPE

Total quality control program at the Louisiana Department of Highways involves analysis of certain problematic highway material characteristics for variability. The program was initiated in the middle of 1963 in cooperation with the Bureau of Public Roads and was broken down into separate phases. Findings of the first two phases have been reported elsewhere.(1, 2) The contents of this report represent the third phase of the study with emphasis on the following portland cement concrete characteristics:

1. Slump
    a. Paving concrete
    b. Structural concrete

2. Gradation of coarse and fine aggregate
3. Thickness of concrete pavement

Concurrently, the following bituminous hot-mix characteristics which were not covered in the first report(1) are also included in this report as a subsection:

(1) Discharge (mixing) temperature
(2) Roadway density

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* Superscript numbers in parenthesis refer to list of references at the end of this report.

OUTLINE OF WORK

Collection of Data

Quality control program necessitates the gathering of vast amount of data. Furthermore, it is almost implicit that these data are unbiased and a religious adherence to random selection of samples is usually necessary to insure this lack of bias. To accomplish this, a specially designed sampling plan using random number tables was used in obtaining data necessary for development of statistical parameters for the characteristics.

Projects were selected on the basis of their geographic locations. For slump of paving and structural concrete, four random trucks were sampled on each project with two replicate determinations per truck. This gave a total of 72 individual observations for each type of concrete (representing nine separate contractors). The tests were performed with different equipment by various operators. For coarse and fine aggregate gradation, two replicate determinations were made on eight randomly selected samples from each separate stockpile (representing each separate source). For fine aggregate, 16 individual tests were performed on each source for a total of 144 observations over the nine sources. For Grade A and B coarse aggregate, a total of 96 and 80 tests were made over the six and five sources respectively. Sampling of the above was performed by different individuals. However, testing was accomplished with one operator using the same equipment.

Data on pavement cores for thickness evaluation was obtained from project files.

The bituminous hot-mix discharge temperatures were obtained using the Bureau of Public Roads'suggested random sampling procedures.(3)

Roadway density data were collected from project files.

Data Processing

All the above data were collected over a period of one year. These raw data were analyzed using IBM 1620 computer and standard statistical procedures.

CHOICE OF ESTIMATORS OF PARAMETERS

Since it is impossible to obtain a true mean and standard deviation of the population, it is necessary to make good estimates of these parameters; that specification limits be closely related to the actual behavior of the process is one of the many reasons for these parameters to be unbiased and efficient.

The mean, $\overline{X}$, is a measure of central tendency of a group of measurements. Mathematically:

$$\overline{X} = \frac{\Sigma X_i}{n} \quad \text{where ,}$$

X_i = individual observations,

and n = number of observations in a group.

The standard deviation, σ (sigma) , is a measure of the dispersion of the measurements from their mean. The mathematical definition is:

$$\sigma = \sqrt{\frac{\Sigma(X_i - \overline{X})^2}{n-1}} \quad \text{where ,}$$

X_i , $\overline{X}$ and n are as above.

The variance, σ^2 , is the square of the standard deviation.

The standard error, $\sigma_{\overline{X}}$, is the standard deviation of the mean of several samples and is estimated by:

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

RELATIONSHIP BETWEEN SPECIFICATIONS AND STATISTICAL PARAMETERS

One of the most commonly used methods of describing pictorially variations of individual observations from within a sample is by means of frequency distribution. In examining data of such type (or any other type), it will be found that the individual observations group themselves about the central value so that there are roughly equal numbers on either side of this central value and small divergencies from this central value occur more frequently than large ones. When this happens, the resulting curve assumes what is termed a Gaussian or Normal Distribution which has a symmetrical bell shape. This is one of the most important distributions in statistics and forms the basis for subsequent analysis of the present data. Its use is the same as that of any other distribution curve-the relative frequency with which a variable will take on values between two points is the area under the curve between the two points on the horizontal axis.

If, instead of plotting individual observations, means of several sample units were plotted, then the resulting distribution would be much narrower. These relationships are shown graphically in Figure 1.

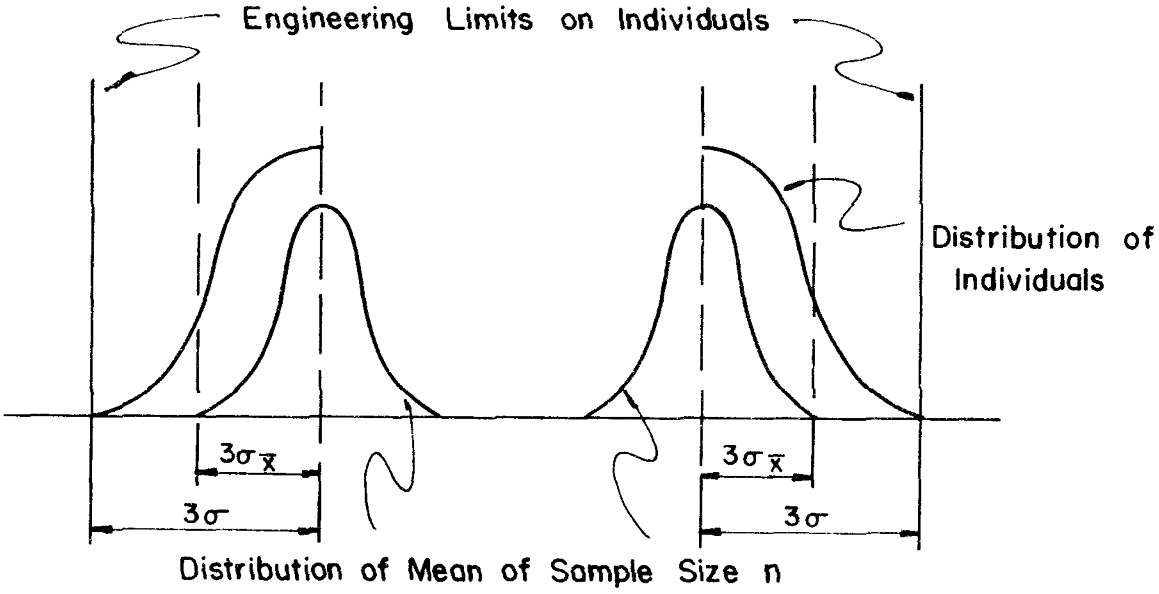


Figure 1: Relationship Between Distribution of Individual Observations and Means of Several Sample Units.

Some of the important characteristics of normal probability curves can be used to represent the accumulated data on different characteristics. If the horizontal axis of this curve is represented by the normal deviate (which is the number of standard deviations of the measurements above or below the mean value), then the area under the normal curve between any two values of the normal deviate (t_1 and t_2) gives the probability that an observation from the population will have a value between t_1 and t_2.

Some of the selected critical values for this normal distribution are shown in Table 21 in the appendix.

The table illustrates that for a normally distributed data, 95.45 percent of the results will be within plus or minus 2σ or that approximately 4.55 percent of the results will be outside the range of $\pm 2\sigma$. Likewise, 99.73 percent will be within $\pm 3\sigma$ and corresponding 0.27 percent outside this range. Thus, knowing the true value of the mean and standard deviation, one can set up limits within which a predetermined proportion of observations shall be included.

Figure 2A symbolizes a relationship between specification tolerance limits and statistical parameters using an idealized normal distribution curve. It indicates great variation with respect to the tolerance limits. This situation is untenable and three solutions are available to modify this situation.

1. Remove the fringe lying outside each tolerance by measuring each and every item which is undoubtedly a costly procedure.

2. Find a new and better method to measure the characteristic (involving research and delay).

3. Revise the limits by making it wider. There is no point in making the specifications so tight they cannot be enforced.

Figure 2B shows a situation where the curve just clears the inside limits. At first, this might seem to be perfect. However, on second thought, there does not seem to be any allowance for operating tolerance and the dotted line shows how the measurements would be outside the limits with only a slight shift in the mean.

The most comfortable situation is illustrated in Figure 2C, where some leeway for sampling, testing, or material variation is allowed. Under this condition, adequate conformance with specification tolerance can be expected.

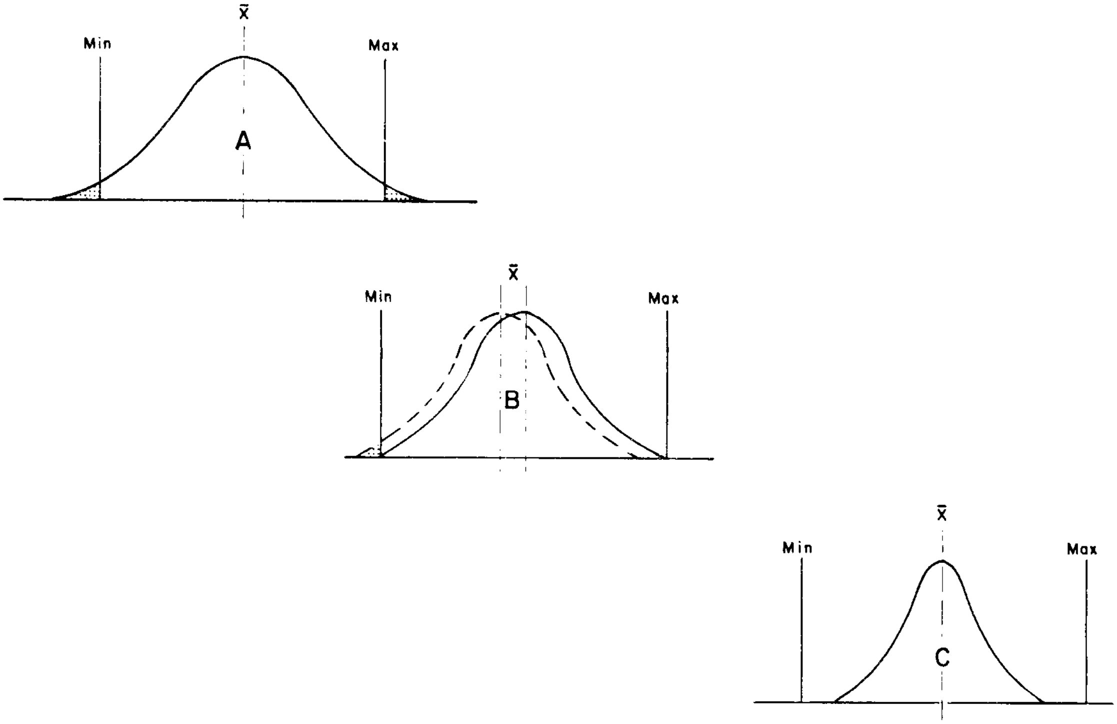


Figure 2: Some Distributional Aspects of Specifications.

To better exemplify the above theoretical relationships, consider the data represented by Figures 3A through 3D which illustrate relationships between currently used specifications and statistical parameters obtained in this study for some of the characteristics. Statistical information is given in Table 1. The figures clearly illustrate that in order for all measurements to conform to the specification limits, the process need to be maintained at the center of the specification limits and the variability of such magnitude as to embrace all the results on either side of the central value ($\pm 3\sigma$). For example, for slump of paving concrete, the process should have been maintained at 2. 25 inches, and the variability equal to or less than 0. 25 inches in order for all measurements to fall inside the current requirements. However, this being not the case, as many as 42 percent of the results failed to meet the specification limits. The reason for such a large number of non conforming results seems obvious.

As mentioned before, three solutions are available to rectify such a situation. If the first two are not economically feasible, then the third could be adopted in which case the revised specifications would be those indicated in the figures. Similar reasoning can be applied to other characteristics and is indicated in the figures.

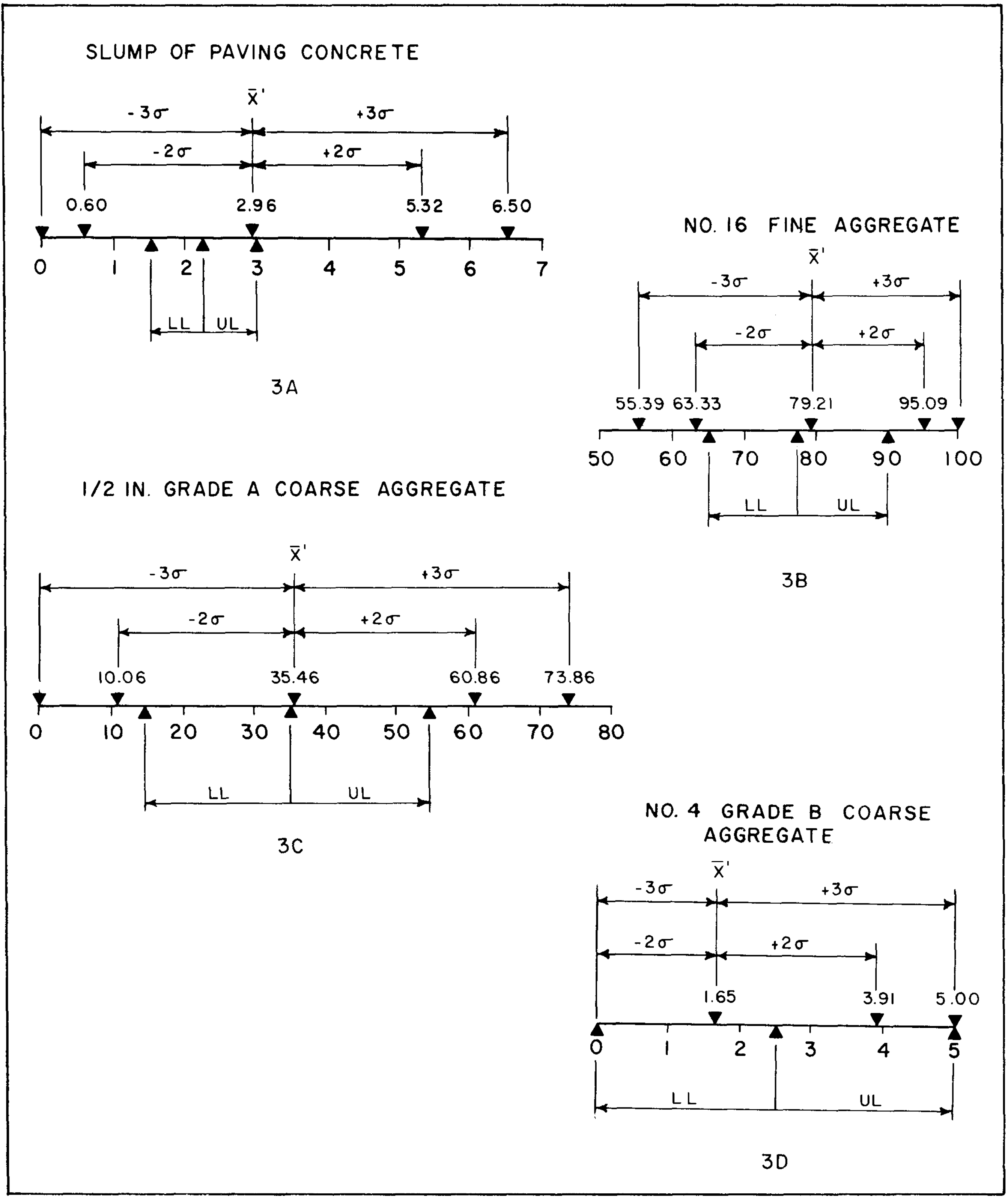


Figure 3: Relationship Between Specifications and Statistical Parameters.

TABLE 1

SUMMARY OF STATISTICAL RESULTS ON SLUMP AND AGGREGATE GRADATIONS

Characteristic	n	$\bar{X}'$	σ^2	σ	Min	Max	% Outside Specs.
Slump of concrete, inches							
Paving	72	2.96	1.39	1.18	0.75	5.75	41.7
Structural	72	2.98	0.77	0.88	1.00	6.00	20.8
Gradation of fine aggregate, percent passing							
No. 4	144	97.82	2.21	1.50	92.1	99.9	2.8
No. 16	144	79.21	62.96	7.94	56.6	91.6	9.7
No. 50	144	15.93	42.71	6.53	7.2	31.6	1.4
No. 100	144	2.06	1.63	1.28	0.3	5.7	0.0
Gradation of grade "A" coarse aggregate, percent passing							
1 in.	96	95.60	14.83	3.85	82.7	99.9	7.3
3/4 in.	96	75.41	117.50	10.84	46.1	88.8	2.1
1/2 in.	96	35.46	162.33	12.70	4.6	60.2	13.5
No. 4	96	1.35	1.48	1.22	0.2	5.5	0.0
Gradation of grade "B" coarse aggregate, percent passing							
3/4 in.	80	73.09	196.56	14.02	23.3	91.8	11.3
No. 4	80	1.65	1.29	1.13	0.3	5.0	0.0

ANALYSIS OF DATA

A number of statistical tools are available for analyzing raw data depending on the kind of data available and the corresponding information desired from such data. Analysis of variance is one such tool which can supply maximum of information from a limited amount of data. Basically, the analysis of variance, hereafter referred to as ANOVA, is just what the name implies--partitioning the variance (i. e. the square of the standard deviation) of an experiment into parts in order to test whether or not certain factors introduced into the design of the experiment actually produce significantly different results in the variable tested.(4) That is, for example, does the different batches of the manufactured concrete or different contractors (sites) affect the measured variable? In otherwords, which of the factors introduced contribute most to the overall variation in the measured characteristic.

Analysis of Variance on Slump of Concrete

Table 2 shows analysis of variance results for slump of paving concrete. The sources of variation, as indicated, are: site-to-site (supplier-to-supplier) variation, trucks-within-site variation (batch-to-batch variation), and subsamples within sample variation which is attributable to experimental error variance.

From the point estimates of the components of variance, it is seen that by far the largest source of variation in results is between trucks within sites. The F test, which is the standard significance test indicates that the site-to-site variation is not significant whereas the trucks-within-site variation is significant at the 0. 05 probability level. This means that the trucks within site component is real and effective.

The error term of 0. 11 indicates that an experimental error of approximately 0. 1 in the slump determination can occur due to chance alone.

In order to compare the magnitude of components of variance from site to site, a one-way ANOVA was also performed for each site separately. The results are presented in Table 3. It is interesting to see the differences in mean square terms from site to site. Site 1 and 9 exhibit the best control as far as this particular test is concerned with Site 4 showing the poorest. The site standard deviation is equal to 0. 53 and 95 percent confidence limits on the site mean will give a range

TABLE 2

ANOVA ON SLUMP OF PAVING CONCRETE

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between sites	33.68	8	4.21	$\sigma_e^2 + 2\sigma_t^2 + 8\sigma_s^2$	(8, 27)NS
Between trucks within sites	60.88	27	2.26	$\sigma_e^2 + 2\sigma_t^2$	(27, 36)*
Between subsamples within samples (Error)	3.94	36	.109	σ_e^2	
Total	98.50	71			

$\sigma_e^2 = .11$ $\sigma_{trucks}^2 = 1.08$ $\sigma_{site}^2 = .24$

NS - Not Significant
* - Significant

TABLE 3

ONE-WAY ANOVA ON SLUMP OF CONCRETE

Site	Mean Squares								
	1	2	3	4	5	6	7	8	9
Paving concrete									
Between trucks	.927	1.527	2.177	5.947	1.653	1.133	3.553	3.147	.230
within samples in trucks	.018	.008	.205	.148	.195	.210	.048	.048	.108
Structural concrete									
Between trucks	3.760	.407	.133	.447	.593	.457	.050	.270	.450
within samples in trucks	.070	.023	.070	.023	.320	.188	.103	.140	.063

within which we are 95 percent confident that the true site mean lies. For example, the mean for Site 1 is 4.10 in. and 95 percent confidence limits would be 3.1 in. to 5.2 in.

The estimated variance of one subsample is equal to 0.109 + 1.08 = 1.19, and the estimated variance of two subsamples from one sample is 0.109/2 + 1.08 = 1.14 which means that there is little gain in the precision of the estimation of the site mean through analyzing two subsamples instead of one. It might, nevertheless, be justifiable to analyze two subsamples as a check against gross errors.

Figure 4 shows distribution of individual test results for all sites. Considerable skewness is observed with several peaks occurring at different points.

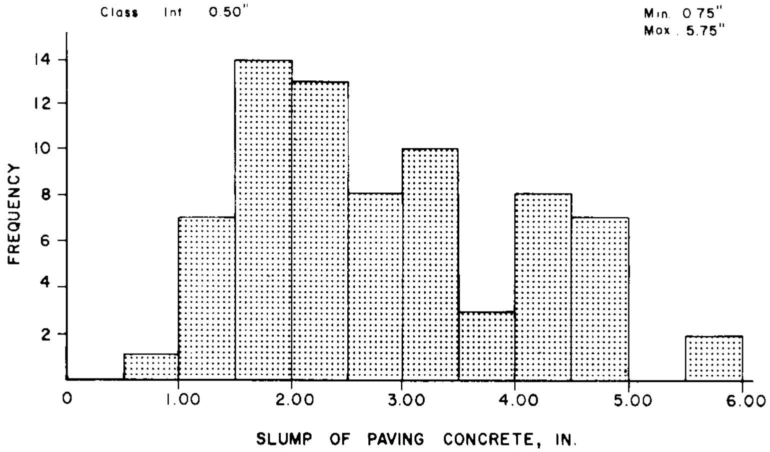


Figure 4: Histogram for the Distribution of Slump of Paving Concrete, All Sites.

The ANOVA for slump of structural concrete is similar to that for paving concrete. The sources of variations and the corresponding mean square terms are indicated in Table 11 in the appendix. In this case, the trucks within site as well as the site component turned out to be significant at the 0.05 significance level. The error term, however, remained the same at 0.11. The site standard deviation of 0.30 in. is considerably less than was indicated for paving concrete. Confidence limits on the site mean indicate that there is 95 percent confidence that the true site mean lies between $\overline{X}_{site}$ ±0.6 in.

Results of one-way ANOVA are indicated in Table 3. Here Site 7 shows maximum control, and Site 1, the least.

The total variance for a single measurement is equal to 0.82 or a standard deviation of 0.90 in.

The distribution of test results for all sites is illustrated in Figure 5. The results plotted are individual test results. The distribution in this case is much closer to normal distribution than was observed for paving concrete.

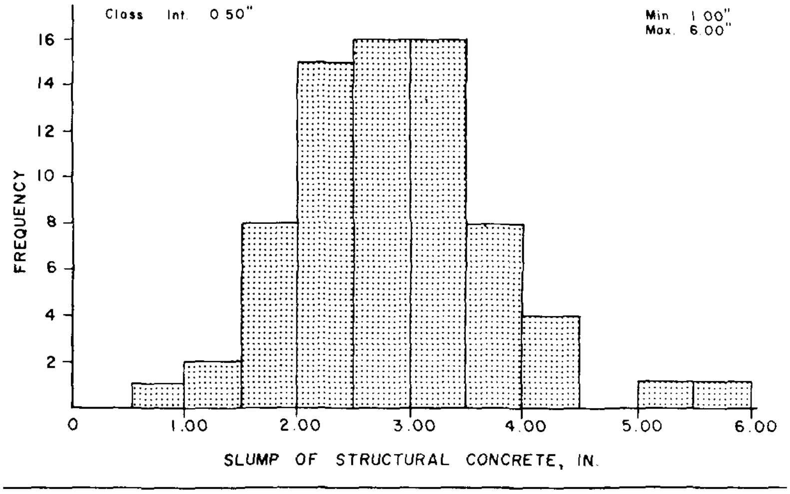


Figure 5: Histogram for the Distribution of Slump of Structural Concrete, All Sites.

The above analysis indicates that there is considerable variation in concrete production from batch to batch as was reflected by larger between trucks-within-site component of variance. This seems obvious if one considers the characteristic of the test where any change in water content from batch to batch will be reflected in the measured variable.

ANOVA on Gradation of Fine Aggregate

Analysis of variance (ANOVA) results for fraction of fine aggregate passing No. 4 sieve is presented in Table 4. Results for other fractions are presented in the appendix. The stockpiles (source of material), samples-within-stockpile and the error term constitute the different components of variance. In each case, it is seen that there is considerable variation between samples within stockpile. The largest component of variance, however, is between stockpiles, which is reflective of material variance (since each stockpile represented different source). The variation between samples within stockpiles can be attributed to either the stockpiling technique or the sampling procedure. The fraction of aggregate passing No. 50 sieve, however, showed the stockpile component to be insignificant. This is due to the fact that the distribution of sample means is so large that it overshadows the distribution among stockpile means (Table 13 in the appendix).

The largest error term was also noted for this fraction. The variance of single measurement is equal to the sum of the point estimates of the components of variance. For example, the total variance for a single measurement for No. 4 sieve is equal to 1.90 + 0.32 + 0.20 = 2.42 of which approximately 79 percent is attributable to differences in stockpile (material), 13 percent due to differences in samples within stockpile (sampling), and the remaining 8 percent to experimental or testing.

Table 5 shows comparison of results of one-way ANOVA from stockpile to stockpile for each aggregate fraction. Note the differences in the mean square term from stockpile to stockpile.

ANOVA on Gradation of Coarse Aggregate

The ANOVA for coarse aggregate gradation is similar to the one for fine aggregate. The sources of variation and the mean square terms for each fraction are shown in Tables 15 through 20 in the appendix. It is interesting to see from these tables the large magnitude of estimated variance for sample-within-stockpile component. This is reverse to that observed for fine aggregate analysis where in the stockpile component was by far the largest of the three. This is because of the larger particle size which is more prone to pronounced segregation particularly if proper stockpiling procedures are not followed. This will further be reflected when the stockpile is sampled and tested.

TABLE 4

ANOVA ON GRADATION OF FINE AGGREGATE
(% Passing No. 4 Sieve)

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between stockpiles	249.43	8	31.18	$\sigma_e^2 + 2\sigma_s^2 + 16\sigma_{st}^2$	(8, 63)*
Between samples within stockpiles	52.44	63	.83	$\sigma_e^2 + 2\sigma_s^2$	(63, 72)*
Between subsamples within samples (Error)	14.35	72	.20	σ_e^2	
Total	316.22	143			

$\sigma_e^2 = .20$ $\sigma_{sample}^2 = .32$ $\sigma_{stockpile}^2 = 1.90$

* - Significant

TABLE 5

ONE-WAY ANOVA ON GRADATION OF FINE AGGREGATE

Stockpile or Source	Mean Squares							
	No. 4		No. 16		No. 50		No. 100	
	BS	WS	BS	WS	BS	WS	BS	WS
1	2.85	.25	76.52	.27	18.40	.18	1.80	.06
2	.04	.08	9.83	.44	.46	.11	.06	.01
3	.78	.11	26.62	.25	7.28	.24	.28	.09
4	.23	.22	27.58	.52	47.74	.12	.43	.02
5	1.04	.31	3.51	.26	19.79	.84	2.96	.02
6	.53	.04	26.11	1.47	43.64	.37	1.07	.05
7	.07	.11	2.14	.17	5.04	.12	.33	.04
8	.48	.14	2.05	.24	615.49	31.91	.37	.03
9	1.49	.61	24.78	2.65	8.23	14.41	.15	.01

BS - Between samples
WS - Within samples

TABLE 6

ONE-WAY ANOVA ON GRADATION OF COARSE AGGREGATE

Stockpile or Source	Mean Squares							
	1 inch		3/4 inch		1/2 inch		No. 4	
	BS	WS	BS	WS	BS	WS	BS	WS
Grade "A"								
1	14.55	.33	448.88	.78	770.59	.45	3.05	.08
2	.87	.87	25.64	1.74	77.28	3.06	.72	.16
3	4.29	1.48	41.86	19.70	67.75	38.97	.60	.36
4	36.63	.70	111.89	3.60	210.70	4.94	4.75	.06
5	16.47	1.67	300.31	8.24	369.91	4.98	.09	.06
6	.01	.01	41.29	4.62	143.29	2.70	.63	.02
Grade "B"								
1	-	-	50.01	2.57	-	-	.10	.19
2	-	-	633.46	1.69	-	-	1.85	.01
3	-	-	107.35	2.44	-	-	4.62	.27
4	-	-	282.53	4.03	-	-	1.70	.40
5	-	-	79.24	7.18	-	-	2.17	.23

BS - Between samples
WS - Within samples

Table 6 compares one-way ANOVA results for the two grades of aggregate from stockpile to stockpile. Again, observable differences occur in the mean square terms from stockpile to stockpile. Note that if the MS terms for the sites are averaged, the resulting average is equal to the corresponding MS term as determined by the ANOVA of the factorial model and provides a check as to the accuracy of the computation procedures.

Thickness of Pavement

Table 7 summarizes statistical findings for pavement thickness. The pooled variance (and hence the standard deviation) for the three thicknesses is approximately the same. The individual values, however, show considerable variation from contractor to contractor.

The histograms, showing the distribution of thickness measurements, are presented in Figure 6. The values plotted are individual thicknesses. The histograms approach approximate normal distribution.

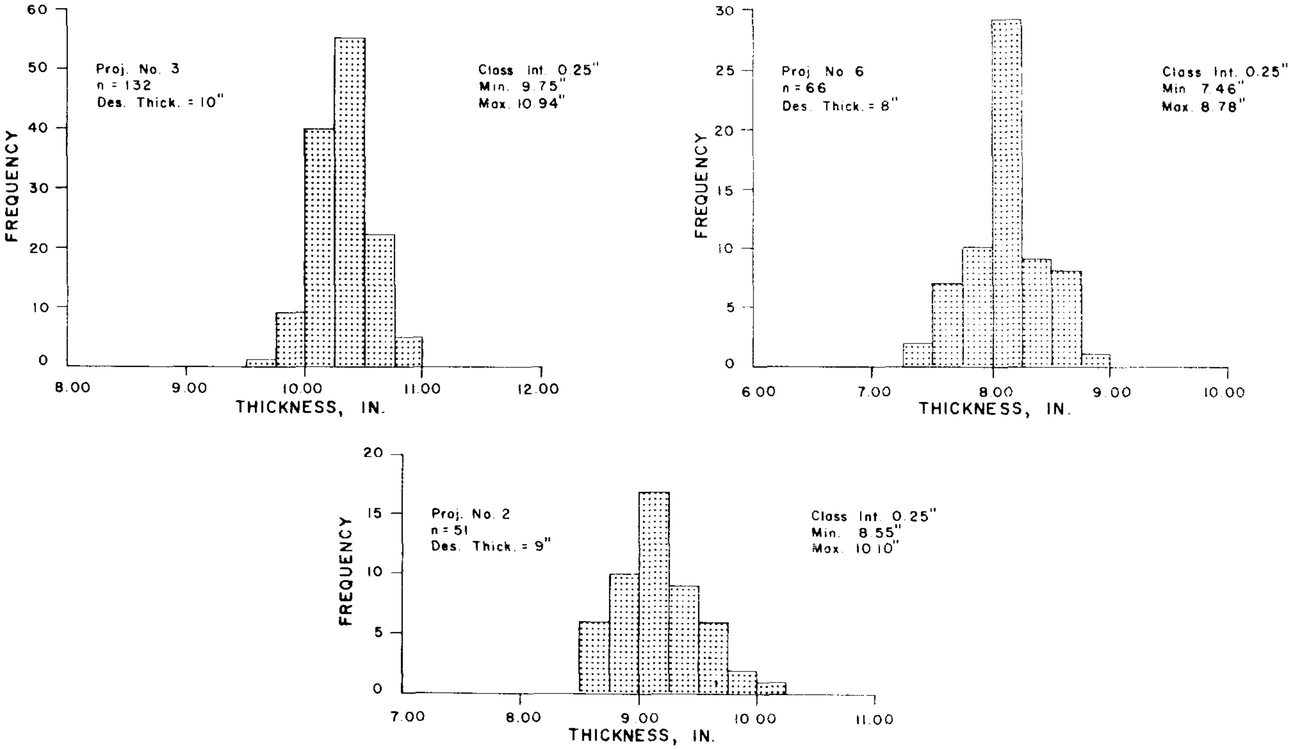


Figure 6: Histogram for the Distribution of Pavement Thickness.

TABLE 7

SUMMARY OF STATISTICAL RESULTS ON THICKNESS OF CONCRETE PAVEMENT

Project Ident.	n	$\bar{X}'$	σ^2	σ	Min.	Max.
8 in. uniform thickness						
1	34	8.66	0.192	0.435	7.63	9.53
2	39	8.42	0.171	0.415	7.61	9.13
3	48	8.35	0.040	0.200	7.86	8.80
4	58	8.36	0.077	0.276	7.76	9.49
5	61	8.05	0.035	0.185	7.66	8.59
6	66	8.11	0.089	0.300	7.46	8.78
7	73	8.06	0.112	0.333	7.58	9.58
Pooled values		8.29	0.088	0.300		
9 in. uniform thickness						
1	35	9.25	0.046	0.210	8.93	9.67
2	51	9.19	0.121	0.350	8.55	10.10
3	58	9.28	0.048	0.220	8.84	9.99
4	65	9.18	0.060	0.240	8.78	9.92
5	74	9.20	0.185	0.430	8.69	11.69
6	88	9.11	0.029	0.170	8.85	9.66
Pooled values		9.20	0.083	0.290		
10 in. uniform thickness						
1	64	10.38	0.061	0.240	9.41	10.91
2	124	10.34	0.079	0.280	9.82	11.48
3	132	10.35	0.079	0.230	9.75	10.94
4	141	10.28	0.083	0.290	9.63	11.27
Pooled values		10.34	0.069	0.270		

ACCEPTANCE SAMPLING BY VARIABLES

Whenever acceptance is to be based on sampling of bulk material such as stockpile of aggregate, batch of concrete or hot-mix, variables sampling plans are most likely to be used. In such plans, the characteristic in question is measured along a continuous scale in terms of pounds per cubic foot, inches, psi, seconds, etc. as opposed to attributes inspection where an item is classified as either defective or non-defective or count the number of defects in it. A practical advantage of using the variable inspection plan is the reduced sample sizes required for specified degrees of protection.

The acceptance function of any inspection must be coupled with a well adopted sampling plan which should specify the following:

1. the size of the sample.
2. the critical value or values of the variable for lot acceptance.
3. the probability of accepting bad products and rejecting good ones.

A strict application of the above variables sampling plan requires rather strong assumption concerning the distribution of the quality characteristic under consideration, viz., that it be normal. The frequency distribution of many measurements is roughly normal, and hence, from practical point of view, this assumption is considered valid.

Knowing the value of the standard deviation for the characteristic, and specified α and β risks, the above $\overline{X}$ single sampling plan may be derived for either single tolerance limit (an upper or lower limit) or double tolerance requiring both an upper and lower tolerance limit from the statistic $\dfrac{\overline{X} - \overline{X}'}{\sigma' / \sqrt{n}}$.

One-Way Protection on Means

Basically, the plan calls for determination of sample size n and acceptance tolerance E and operates as follows:

1. Select a random sample of size n from the lot.
2. Find $\overline{X}$, the mean of this lot.
3. If $\overline{X}$ is greater than or equal to some value K, we accept the lot; otherwise, reject it or take corrective action.

To any sampling plan, we are required to associate what is called the producer's and consumer's risk. In acceptance sampling, there is always a chance that one may erroneously reject a good lot which would be a sacrifice for the producer or the contractor. This is the producter's risk (α). On the otherhand, one may accept a bad lot erroneously which would be a risk for the consumer or the State. This is the consumer's risk (β). What can be considered satisfactory risks depends on the criticality of the variable and the economics of sampling and testing. However, if the variable is considered critical enough as to affect the successful performance of the end product, then the risks should be adopted accordingly. It is believed that for a major characteristic, the producer's risk can be set at 0. 02 and the consumer's risk at 0. 05. $\alpha = 0.02$ means that the probability of rejecting lots of acceptable mean quality is 0. 02. Likewise, $\beta = 0.05$ means that 5 percent of the time bad lots would be accepted if offered by the contractor. This is shown graphically in Figure 7.

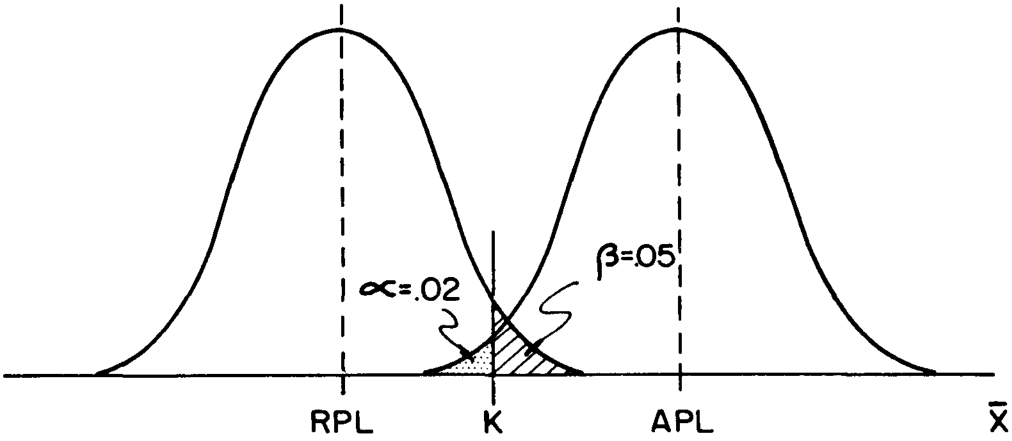


Figure 7: Distributions of $\bar{x}$ for Acceptable and Rejectable Process Levels and the Corresponding Risks (One-way Protection).

The simultaneous solution of two equations containing standarized normal deviate associated with α and β will give the desired sample size and acceptance limits.(2, 5)

Two-Way Protection on Means

The above plan was for variables requiring only one-way protection. Similar reasoning can be applied to variables requiring two-way protection.

Again assuming normal distribution of sample means and the values of α and β as before, the problem can be illustrated graphically thus:

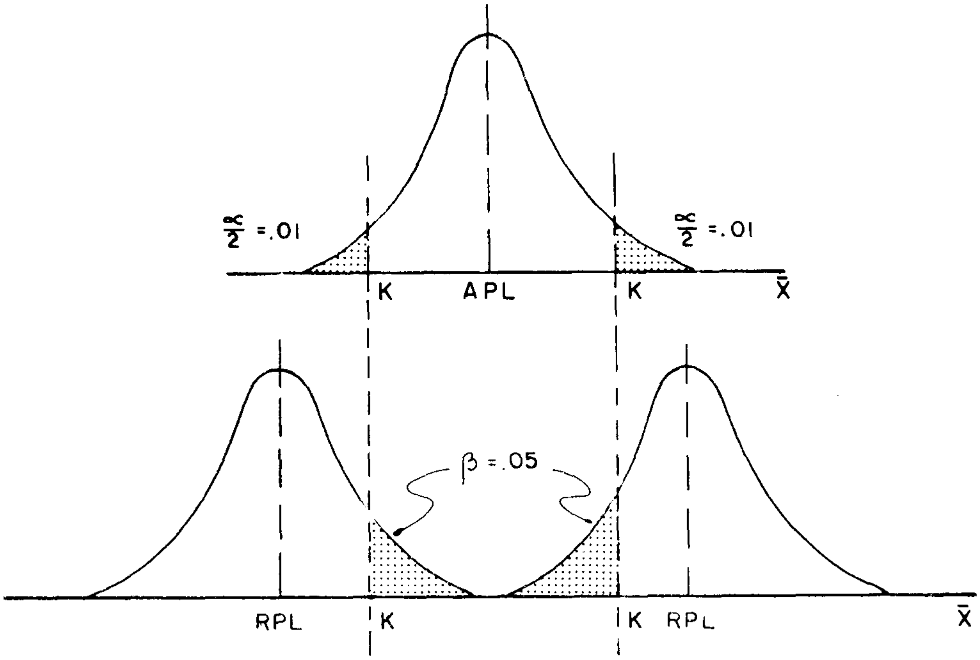


Figure 8: Distributions of $\bar{x}$ for Acceptable and Rejectable Process Levels and the Corresponding Risks (Two-way Protection).

Protection on Individuals

A disadvantage in using acceptance sampling for means of variables is that the mean of the sample may conform to some set value regardless of the individual values in the sample which may be too low and the rest sufficiently high to give the desired average. To safeguard against this, limits on individuals should also be enforced along with those on the means.

In the preceding section where the size of the sample has already been established for certain risks, it is only necessary to use criteria which can tell whether any single measurement from the sample could be considered to be from population other than that sampled. This criteria for outliers is explained in ASTM E 178. Reference (6) treats this subject in more detail.

The above single sampling plans were based on the statistic $\frac{\bar{X} - \bar{X}'}{\sigma' / \sqrt{n}}$ where σ is assumed to be known. However, when the value of standard deviation is not known, the statistic $\frac{\bar{X} - \bar{X}'}{R}$ can also be used, R being the range or the difference between the maximum and minimum value in the group.

Table 8 summarizes the suggested acceptance sampling plan described in the preceding section. The sample size n indicated is not a limiting value but a definite one and any change in the number will affect the corresponding risks and hence the tolerances.

TABLE 8

SUGGESTED ACCEPTANCE LIMITS

Probability of Acceptance P_a	Probability of Rejection P_r	Sample size n	Acceptance Limits	
			Means $\bar{X}' \pm T_s$	Individuals $\bar{X}_s \pm T_i$
98	95	5	$\bar{X}' - .92\sigma$	$\bar{X}_s \pm 2.48\sigma$
98	95	5	$\bar{X}' \pm 1.04\sigma$	$\bar{X}_s \pm 2.48\sigma$
99	90	4	$\bar{X}' \pm 1.29\sigma$	$\bar{X}_s \pm 2.62\sigma$

ASPHALTIC CONCRETE

Table 9 shows summary of statistical results for bituminous hot mix roadway density and discharge temperature. As mentioned under Outline of Work, density data represent laboratory-run specimens and were obtained from project files. Note the differences in standard deviations from project to project.

The analysis of variance results for discharge temperature indicate contribution of material variance to be the largest. This seems obvious since, aggregate temperatures and bituminous temperatures, both affect the final discharge or mixing temperature which is further reflected from batch to batch and day to day production. It is believed, that a much closer control can be maintained to reduce this process variability.

TABLE 9

SUMMARY OF STATISTICAL PARAMETERS FOR ASPHALTIC CONCRETE

Project Iden.	n.	$\bar{X}'$	σ_a^2	σ_t^2	σ^2	σ	Min.	Max.
Roadway density (percent of laboratory).								
A	60	98.14	-	-	1.24	1.11	94.1	99.5
B	92	98.81	-	-	1.06	1.03	95.7	102.0
C	110	96.60	-	-	3.04	1.74	93.7	102.8
D	135	98.26	-	-	2.10	1.45	90.7	100.9
E	138	97.57	-	-	1.61	1.28	94.6	100.4
F	219	97.89	-	-	2.48	1.58	89.6	100.8
G	252	97.01	-	-	3.40	1.84	90.5	103.0
Pooled values		97.64			2.76	1.66		
Discharge (mixing) temperature, deg fahr.								
A	200	324.03	128.84	9.44	138.28	11.76	280	370
B	200	316.23	253.02	5.61	258.63	16.08	275	350
C	200	316.43	119.30	1.53	120.83	10.99	285	360

CONTROL CHARTS

Control of any repetitive process such as production of portland cement concrete or bituminous hot-mix is of prime importance. A control chart is a statistical tool, which gives a visual indication of the state of control of any production process. It is an instrument to be used in specification, production, and inspection and when so used, brings these three phases of industry into an interdependent whole.

In any production process, two types of variations are known to exist, one belonging to the category of chance variations about which little can be done since they are not identifiable, and the other produced by "assignable causes" which are relatively large variations attributable to some certain causes which are identifiable. For instance, the quality of output of the morning shift may differ from that of the evening shift and quality of output of plant A may differ from that of plant B even under identical operating conditions. But, though we may account for the variability between shifts and between plants, there still remains variability of a random nature within shifts and within plants. Thus, when all the non-random type of variations are eliminated or taken into consideration and the probability distributions of the random or chance variation has been discovered, the process is said to be in a state of control.

The variables control charts most commonly used are average or $\overline{X}$ - charts and range or R - charts. An $\overline{X}$ - chart shows variation in the averages of samples. On it are drawn the central line and upper and lower control limits.

A sample of n items is taken from the process at some interval of time or quantity, and after determining the quality measurement, the average of these n items is plotted on this chart. As long as the points fall within the band, the process is considered to be in control. If a point falls outside the control limits, the process is said to be out of control and an investigation is made to find the assignable cause of this variation. The R - charts are similar except that instead of means of n items, the ranges are plotted and any point outside the band indicates lack of control with respect to the variability of the process.

Sometimes the charts can be used as dual purpose charts, viz. , to control current production and also for acceptance of the lot. The limits on such a chart could be those computed in the previous section (Table 8). Figure 9 illustrates the use of such charts for slump of structural concrete. The limits are those shown in Table 10. In order to illustrate the dual purpose of these charts, the slump measurements were arranged consecutively in subgroups of five. Interesting observations can be made from these charts. For example, chart for means indicates Lot 1 to have slump values close to the upper limit which suddenly drops to the central value for Lot 2 and keeps on decreasing to a value where it barely meets the lower tolerance limit for Lot 5. Adjustments were made to bring the process in control as is indicated for Lot 6.

If a point falls outside the limits on the $\bar{X}$ - chart, the chart for individuals will show whether the shift in the mean was due to one value or the whole group. This is illustrated by Lot 1 on the chart for individuals. The middle chart represents control chart for ranges and illustrates the shift of variability from lot to lot.

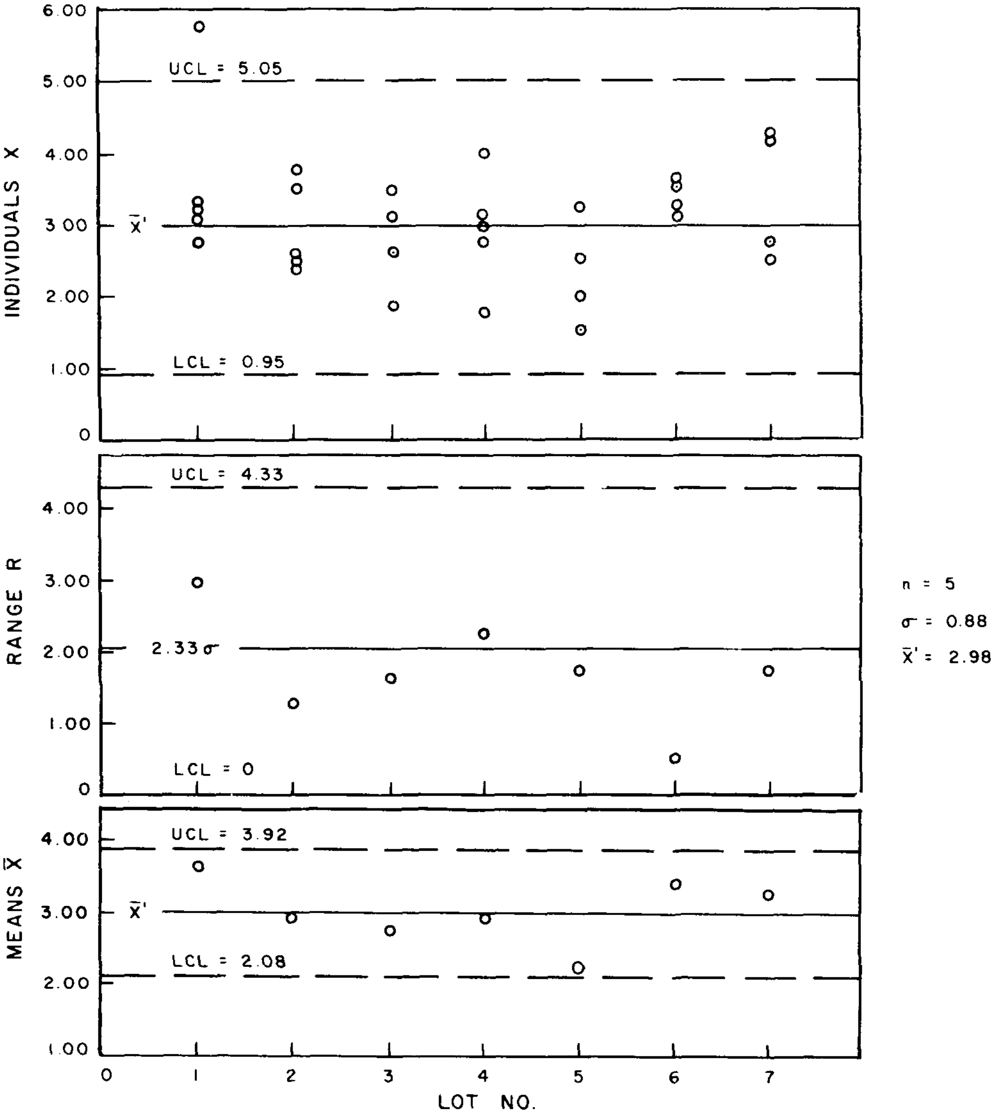


Figure 9: Control charts for Acceptance and Control of Current Production (Data on Slump of Structural Concrete).

RECOMMENDATIONS

This report represents the third and last phase of analysis of some of the problematic highway construction materials characteristics for variability, the purpose being to evolve realistic specification limits. The findings of the first two phases have been reported elsewhere. (1, 2)

The best of specifications can not fully accomplish their objectives unless they are uniformly interpreted and enforced. However, uniformity in interpretation and enforcement can only be achieved through proper knowledge of what is involved in statistically adopted specifications. The contractor who is familiar with the rudiments of statistical fundamentals pertaining to acceptance plans will have little to complain, since, he will be aware that the limits are based on normal patterns of variation encountered in normal production or construction process and, that it will be uniformily applied to the rest of his competitors. Likewise, the technician who has been schooled on these fundamentals and the working of the acceptance plans, will not waver in rejecting a sample or a lot when it is on the borderline. This will certainly minimize the "take one more sample" trend.

Adoption of statistically derived acceptance plans for material or job compliance will undoubtedly present multitude of factors, some large as to involve administrative decisions and others small enough to be tackled by field personnel. Any effort to list these would be a report in itself. However, it is believed that the suggestions listed below would enhance implementation of the overall program.

1. Set up some sort of educational program for the Department's personnel to expose them to the rudiments of statistical acceptance plans. To accomplish this may require preparation of manual with emphasis on:

   (a) Definition of terms that are easily understood limiting the mathematics to an absolute minimum.

   (b) Frequency distribution and its use.

   (c) Measures of central tendency and variability.

   (d) Control charts.

   (e) Acceptance sampling by variables and use of random-numbers table.

   (f) Application of the above to highway materials as covered in the three reports.

2. Select three different projects for each of the major material characteristics (soils, concrete and asphaltic concrete) to check the effectiveness of the statistically derived acceptance sampling plan. The acceptance limits can be incorporated in the contract specifications as special provisions. Furthermore, material characteristics not covered in this study and for which statistical parameters are not known, should also be included in this phase using appropriate sampling plans. This entire phase can constitute Phase IV of the current project.

3. Whenever production process is involved such as manufacture of concrete or bituminous hot mix, control charts should be used. Such charts, in addition to serving as tool to control current production will also serve as a permanent record of production, sampling, testing and acceptance.

4. Acceptance specifications work better when a number of individual properties are expressed as a whole. For example, gradation of aggregate could be expressed in terms of a single parameter instead of a multiplicity of percentages.

Summing up this report, the correctness of any specification is determined by the mathematical relationship between use and production coupled with cost. Specifications fundamentally should be based on facts and these must be known first. They must come from a study of the product, of its use, and its production. Last but not least, the correct specification mean an open, cooperative effort by the producer and consumer. Each must be conversant with the problems of the other and both must be willing to study the over-all problem.

REFERENCES CITED

(1) Verdi Adam and S. C. Shah, "Quality Control Analysis of Asphaltic Concrete," Proceedings, Highway Conference on Research and Development of Quality Control and Acceptance Specifications, Vol. 1, U. S. Department of Commerce, Bureau of Public Roads, April, 1965.

(2) S. C. Shah, "Quality Control Analysis-Part II-Soil and Aggregate Base Course," Louisiana Department of Highways, Research Report No. 23, July, 1966.

(3) "The Statistical Approach to Quality Control in Highway Construction," U. S. Department of Commerce, Bureau of Public Roads, April, 1965.

(4) C. R. Hicks, "Fundamentals of Analysis of Variance," Part I, Industrial Quality Control, August, 1956, p. 17.

(5) A. J. Duncan, Quality Control and Industrial Statistics, Richard D. Irwin, Inc., Illinois, 1959, pp. 244-247.

(6) H. A. David, "Revised Upper Percentage Points of the Extreme Studentized Deviate from the Sample Mean," Biometrika, Vol. 43, Parts 3 and 4, 1956, pp. 449-51.

(7) W. A. Shewhart, Statistical Method from the Viewpoint of Quality Control, Graduate School, Department of Agriculture, Washington, 1939, Chap. i.

APPENDIX

TABLE 11

ANOVA ON SLUMP OF STRUCTURAL CONCRETE

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between sites	31.17	8	3.90	$\sigma_e^2 + 2\sigma_t^2 + 8\sigma_s^2$	(8, 27)*
Between trucks within sites	19.70	27	.73	$\sigma_e^2 + 2\sigma_t^2$	(27. 36)*
Between subsamples within samples (Error)	4.00	36	.111	σ_e^2	

Total	54.85	71

$\sigma^2 = .111$ $\sigma^2_{truck} = .31$ $\sigma^2_{site} = .40$

* - Significant

TABLE 12

ANOVA ON GRADATION OF FINE AGGREGATE
(% Passing No. 16 Sieve)

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between stockpiles	7560. 09	8	945. 11	$\sigma_e^2 + 2\sigma_s^2 + 16\ \sigma_{st}^2$	(8, 63)*
Between samples within stockpiles	1393. 61	63	22. 12	$\sigma_e^2 + 2\sigma_s^2$	(63, 72)*
Between subsamples within samples (Error)	50. 25	72	. 70	σ_e^2	
Total	9003. 95	143			

$\sigma_e^2 = .70$ $\sigma_{sample}^2 = 10.72$ $\sigma_{stockpile}^2 = 57.69$

* - Significant

TABLE 13

ANOVA ON GRADATION OF FINE AGGREGATE
(% Passing No. 50 Sieve)

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between stockpiles	359.74	8	44.97	$\sigma_e^2 + 2\sigma_s^2 + 16\sigma_{st}^2$	(8, 63)NS
Between samples within stockpiles	5362.17	63	85.11	$\sigma_e^2 + 2\sigma_s^2$	(63, 72)*
Between subsamples within samples (Error)	386.27	72	5.36	σ_e^2	
Total	6108.18	143			

$\sigma_e^2 = 5.36$ $\sigma_{sample}^2 = 39.88$ $\sigma_{stockpile}^2 = 0$

NS - Not Significant
* - Significant

TABLE 14

ANOVA ON GRADATION OF FINE AGGREGATE
(% Passing No. 100 Sieve)

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between stockpiles	178.23	8	22.28	$\sigma_e^2 + 2\sigma_s^2 + 16\sigma_{st}^2$	(8, 63)*
Between samples within stockpiles	52.01	63	.83	$\sigma_e^2 + 2\sigma_s^2$	(63, 72)*
Between subsamples within samples (Error)	2.66	72	.037	σ_e^2	
Total	232.90	143			

$\sigma_e^2 = .037$ $\sigma_{sample}^2 = .395$ $\sigma_{stockpile}^2 = 1.34$

* - Significant

TABLE 15

ANOVA ON GRADATION OF GRADE "A" COARSE AGGREGATE
(% Passing 1-inch Sieve)

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between stockpiles	858.67	5	171.73	$\sigma_e^2 + 2\sigma_s^2 + 16\sigma_{st}^2$	(5, 42)*
Between samples within stockpiles	509.69	42	12.14	$\sigma_e^2 + 2\sigma_s^2$	(42, 48)*
Between subsamples within samples (Error)	40.52	48	.84	σ_e^2	
Total	1408.88	95			

$\sigma_e^2 = .84$ $\sigma_{sample}^2 = 5.65$ $\sigma_{stockpile}^2 = 9.97$

* - Significant

TABLE 16

ANOVA ON GRADATION OF GRADE "A" COARSE AGGREGATE
(% Passing 3/4-inch Sieve)

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between stockpiles	4065.47	5	813.09	$\sigma_e^2 + 2\sigma_s^2 + 16\ \sigma_{st}^2$	(5, 42)*
Between samples within stockpiles	6789.08	42	161.65	$\sigma_e^2 + 2\sigma_s^2$	(42, 48)*
Between subsamples within samples (Error)	307.68	48	6.41	σ_e^2	
Total	11,162.23	95			

$\sigma_e^2 = 6.41$ $\sigma^2_{\text{sample}} = 77.62$ $\sigma^2_{\text{stockpile}} = 40.72$

* - Significant

TABLE 17

ANOVA ON GRADATION OF GRADE "A" COARSE AGGREGATE
(% Passing 1/2-inch Sieve)

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between stockpiles	3504.26	5	700.85	$\sigma_e^2 + 2\sigma_s^2 + 16\sigma_{st}^2$	(5, 42)*
Between samples within stockpiles	11,476.69	42	273.26	$\sigma_e^2 + 2\sigma_s^2$	(42, 48)*
Between subsamples within samples (Error)	440.74	48	9.18	σ_e^2	
Total	15,421.69	95			

$\sigma_e^2 = 9.18$ $\sigma_{sample}^2 = 132.04$ $\sigma_{stockpile}^2 = 26.72$

* - Significant

TABLE 18

ANOVA ON GRADATION OF GRADE "A" COARSE AGGREGATE
(% Passing No. 4 Sieve)

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between stockpiles	65.81	5	13.16	$\sigma_e^2 + 2\sigma_s^2 + 16_{st}^2$	(5, 42)*
Between samples within stockpiles	68.83	42	1.64	$\sigma_e^2 + 2\sigma_s^2$	(42, 48)*
Between subsamples within samples (Error)	5.88	48	.13	σ_e^2	
Total	140.52	95			

$\sigma_e^2 = .13$ $\sigma_{sample}^2 = 0.76$ $\sigma_{stockpile}^2 = .72$

* - Significant

TABLE 19

ANOVA ON GRADATION OF GRADE "B" COARSE AGGREGATE
(% Passing 3/4-inch Sieve)

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between stockpiles	7177.10	4	1794.28	$\sigma_e^2 + 2\sigma_s^2 + 16\sigma_{st}^2$	(4, 35)*
Between samples within stockpiles	8208.00	35	234.51	$\sigma_e^2 + 2\sigma_s^2$	(35, 40)*
Between subsamples within samples (Error)	143.20	40	3.58	σ_e^2	
Total	15,528.30	79			

$\sigma_e^2 = 3.58$ $\sigma_{\text{sample}}^2 = 115.47$ $\sigma_{\text{stockpile}}^2 = 97.49$

* - Significant

TABLE 20

ANOVA ON GRADATION OF GRADE "B" COARSE AGGREGATE
(% Passing No. 4 Sieve)

Source of Variance	SS	df	MS	EMS	$F_{.05}$
Between stockpiles	21.37	4	5.34	$\sigma_e^2 + 2\sigma_s^2 + 16\sigma_{st}^2$	(4, 35)NS
Between samples within stockpiles	73.05	35	2.09	$\sigma_e^2 + 2\sigma_s^2$	(35, 40)*
Between subsamples within sample (Error)	8.72	40	.22	σ_e^2	
Total	103.14	79			

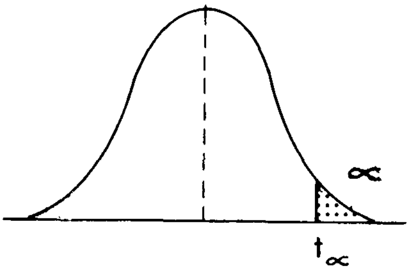
$\sigma_e^2 = .22$ $\sigma_{sample}^2 = .94$ $\sigma_{stockpile}^2 = .20$

NS - Not significant
* - Significant

TABLE 21

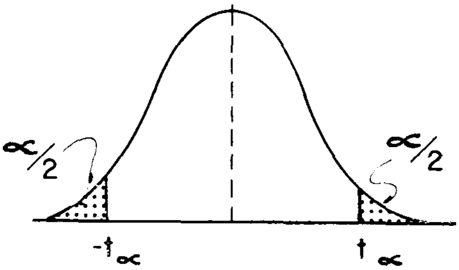
SELECTED VALUES FOR THE NORMAL DISTRIBUTION

One Tail Critical Values



α or β	t_α or t_β
.10	1.282
.05	1.645
.0455	1.690
.025	1.960
.02	2.054
.01	2.326
.005	2.576
.0027	2.782
.002	2.878
.001	3.090

Two Tail Critical Values



α or β	t_α or t_β
.10	1.645
.05	1.960
.0455	2.000
.025	2.241
.02	2.326
.01	2.576
.005	2.807
.0027	3.000
.002	3.090
.001	3.291

SYMBOLS

E - The specification tolerance which determines acceptance limits.

K - The desired value for acceptance of the lot.

LCL - Lower control limit on control charts.

LL - Lower specification limit.

n - The number of observations in a group or subgroup.

P_a - Probability of accepting good material having the desired average value.

P_r - Probability of rejecting bad material having the lowest acceptable average.

R - The range or the difference between the maximum and minimum measurement in a set of data.

t - The normal deviate or the number of standard deviations of the measured characteristic above or below the mean value as measured on the horizontal axis.

UCL - Upper control limit on control charts.

UL - Upper specification limit.

X_i - The value of a single measured characteristic.

$\bar{X}$ - The average or arithmetic mean found by dividing the sum of n observations by the number of observations.

$\bar{X}_s$ - The average of a number of measurements in a sample.

$\bar{X}'$ - The universe mean or the true average quality of the measured characteristic.

σ - (Sigma) - The Standard deviation which is a measure of the dispersion of a group of measurements from their average.

α	- (Alpha) - The Producer's risk or the probability of rejecting lots of acceptable mean quality.
β	- (Beta) - The Consumer's risk or the probability of accepting lots of rejectable mean quality.
Σ	- A symbol for summation of values.
σ_a^2	- The inherent process variance.
σ_t^2	- The variance due to test method.

RESEARCH PUBLICATIONS

1. Concrete Pavement Research. H.L. Lehmann and C.M. Watson, Part I (1956) , Part II (1958) .

2. Use of Self-Propelled Pneumatic-Tired Rollers in Bituminous Construction and Recommended Procedures. A Special Report, 1958.

3. Use of Expanded Clay Aggregate in Bituminous Construction. H.L. Lehmann and Verdi Adam, 1959.

4. Application of Marshall Method in Hot Mix Design. Verdi Adam, 1959.

5. Effect of Viscosity in Bituminous Construction. Verdi Adam, 1961.

6. Slab Breaking and Seating on Wet Subgrades with Pneumatic Roller. J.W.Lyon, Jr.,January, 1963.

7. Lightweight Aggregate Abrasion Study. Hollis B. Rushing, Research Project No. 61-7C, February, 1963.

8. Texas Triaxial R-Value Correlation. Harry L. Roland, Jr., Research Project No.61-1S, March, 1963.

9. Asphaltic Concrete Pavement Survey. S.C.Shah,Research Project No. 61-1B, April, 1963.

10. Compaction of Asphaltic Concrete Pavement with High Intensity Pneumatic Roller, Part I. Verdi Adam, S.C. Shah and P.J. Arena, Jr., Research Project No.61-7B, July, 1963.

11. A Rapid Method of Soil Cement Design. Harry L. Roland, Jr., Ali S. Kemahlioglu, Research Project No.61-8S, March, 1964.

12. Correlation of the Manual Compaction Hammer with Mechanical Hammers for the Marshall Method of Design for Asphaltic Concrete. P.J.Arena, Jr., Research Project No. 63-1B, September, 1964.

13. Nuclear Method for Determining Soil Moisture and Density. Harry L. Roland, Jr., Research Project No. 62-1S, November, 1964.

14. Service Temperature Study for Asphaltic Concrete. P.J. Arena, Jr., Research Project No.61-3B, October, 1964

15. Quality Control Analysis, Part I-Asphaltic Concrete. S.C. Shah, Research Project No. G3-1G, November, 1964.

16. Typical Moisture--Density Curves. C.M. Higgins, Research Project No.61-11S, May, 1965.

17. High Pressure Lime Injection. C.M. Higgins, Research Project No. 63-7S, August, 1965.

18. Durability of Lightweight Concrete-phase 3. Hollis B. Rushing, Research Project No. 61-8C,August, 1965.

19. Compaction of Asphaltic Concrete Pavement with High Intensity Pneumatic Roller, Part 11-Densification Due to Traffic. S.C. Shah, Research Project No. 61-7B, October, 1965.

20. A Rapid Method for Soil Cement Design-Louisiana Slope Value Method Part 11 Evaluation. C.M. Higgins,A.S. Kemahlioglu, Verdi Adam Research Project No. 61-8S, May, 1966.

21. Typical Moisture-Density Curves, Part II-Lime Treated Soils. C.M. Higgins, Research Project No. 61-11S, May 1966

22. Nuclear Moisture-Density Evaluation, Part 11. C.M. Higgins, Research Project No. 62-1SB, May, 1966

23. Quality Control Analysis, Part II-Soil and Aggregate Base Course, S.C.Shah, Research Project No. 63-1G, May, 1966.

24. Quality Control Analysis, Part III-Concrete and Concrete Aggregate. S.C. Shah, Research Project No. 63-1G, November, 1966